Report to BC Equivalency Committee
MAT from CSUB vs. Master's in Math Ed.

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November 17, 2022

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Comparison Report

Background:

A graduate degree in mathematics consists (minimally) of three core areas of study:

1. Real Analysis
2. Complex Analysis
3. Algebra (abstract)

Many programs include a 4th area:

4. Topology

As articulated in the attached paper by Dr. Fredenberg, a graduate degree in mathematics education will include some graduate-level content knowledge.

Key Findings:

The MAT from CSU Bakersfield is devoid of any graduate-level mathematics while the Master's in Math Education degrees from CSUSD and CSULB ***require*** at least some graduate-level math. The MAT also lacks parity with the Education degree within the domain of education coursework.

	Number of Ed Courses	**Undergraduate Topics Math Courses**	**Graduate Topics Math Courses**
CSUB	**3**	**7**	**0**
CSUSD	**7**	**3**	**3**
CSULB	**5+**	**7*/0-8+**	**1-9+**

***Indicates prerequisites**

*The 7 undergraduate courses are pre-requisites. CSULB requires 9 math courses with at least one at graduate-level. However, any undergraduate upper division courses must be approved.

Summary:

The MAT from CSUB lacks the education coursework AND mathematical content found in other CSU's Master's of Math Education.

CSUB Graduate Program

This degree is designed for working mathematics teachers with a Single Subject Credential in Mathematics (from California or other state), or mathematics teachers holding a Supplementary Authorization in Elementary Mathematics to a Single or Multiple Subject Credential. The purpose of the Master of Arts in Teaching Mathematics is to enable secondary and middle school mathematics teachers to deepen their mathematical and pedagogical understanding of the mathematics of secondary school. The course of study is designed to deepen the participant's mathematical knowledge and integrate it with his/her prior experiences and training.

MATH 520: Discrete Mathematics Models

5.00 Credits Construction and analysis of difference models from physical, biological, social, and financial sciences. Cobweb analysis, convergence, stability and chaos in discrete dynamical systems. Phase plane analysis of systems of difference equations. (5 units lecture)

MATH 521: Statistics and Data Analysis

5.00 Credits Classical and non-parametric confidence intervals and hypothesis tests for means, proportions, linear and multiple regression, and analysis of variance. Techniques include graphical displays, transformations, outlier identification, smoothing, regression and robustness; use of statistical package. Prerequisite: prior experience with statistical analysis at the level of Math 140. (5 units lecture)

MATH 522: Num Approach to Calc & Diff Equations

5.0 credits Use of numerical and algebraic techniques to study change. The use of forward, back, and symmetric differences in data analysis. Divided differences as average rates of change and as approximations to instantaneous rate of change. Difference equations and Euler's method as numerical approximations to differential equations. Riemann sums, midpoint, trapezoid and Simpson's method to approximate accumulated change. Error analysis for numerical approximations. Prerequisite: (1) MATH 520 or (2) MATH 300. (5 units lecture)

MATH 523: Geometric Linear Algebra

5.00 Credits Algebra and geometry of vectors in two and three dimensions. Complex numbers. Matrices as linear transformations of R2 and R3. Rigid motions in two and three dimensions, their classification and matrix representations: Theory and classification of conic sections. Theory of perspective. Prerequisite: MATH 300. (5 units lecture)

MATH 524: Number Theory and Cryptography

5.00 Credits Elementary theory of natural numbers, including prime numbers, divisibility, modular arithmetic, and Diophantine equations. Applications in cryptography such as Caesar ciphers and RSA cryptosystem. Additional topics selected by instructor. This course makes use of a significant amount of calculator or computer programming. Prerequisite: MATH 300. (5 units lecture)

MATH 525: Dynamical Geometry

5.00 Credits Investigations in the Euclidean geometry of two dimensions using dynamical software. Emphasis on exploration, conjecture and verification. Prerequisite: MATH 300 or equivalent experience with the role of proof in Mathematics. (5 units lecture)

MATH 526: History of Mathematics

5.00 Credits Development of mathematics from its empirical origins to its present form. Euclid's Elements. Prerequisites: (1) MATH 300, and (2) at least two mathematics courses that require MATH 300 as a prerequisite. (4 units lecture and 1 units lab)

MATH 540: Intro to Math Educ Research

5.00 Credits Brief introduction to basic philosophies, key terms and generally accepted strategies of both quantitative and qualitative research, such as the criteria and procedures appropriate for establishing validity reliability, credibility, and trustworthiness. Understanding ethics, confidentiality, protection of human subjects, variables, sampling, and data collection. Major emphasis on being able to find, evaluate, and use research in math education. (5 units lecture)

MATH 577: Advanced Topics

1.00 Credits Topics and prerequisites to be announced. May be repeated for different topics. General prerequisite: Major or minor in Mathematics. (1-5 units lecture)

MATH 580: Adv Research Participation

1.00 Credits Individual mathematical investigation, under supervision. (Experience as a research assistant does not count for credit.) Offered on a credit/no credit basis only. Prerequisite: Permission of instructor. (1-5 units lecture)

MATH 591: Culminating Project

5.00 Credit Design and implementation of a written report of mathematical, or field research or similar activity. Prerequisites: Successful completion of 30 approved credits towards the Master of Arts in Mathematics for Teachers. Appointment of a Culminating Activity Committee consisting of three faculty members approved by the Mathematics Graduate Program Committee. (5 units lecture)

CSUSD Graduate Program

Upper Division Courses
Mathematics Courses for Mathematics Education

MATH 501A. Reasoning: Place Value and Arithmetic Operations (1)
Prerequisites: *Teaching credential and consent of instructor.*
Place value and its role in development and understanding of arithmetic operations, to include numeration systems, student methods, standard algorithms, and mental computation.

MATH 501B. Reasoning: Rational Numbers and Real Number Systems (1)
Prerequisites: *Teaching credential and consent of instructor.*
Rational numbers and structure of real number system, to include meanings and models for fractions with attention to operations on rational numbers.

MATH 504A. Reasoning: Quantities and Mathematical Relationships (1)
Prerequisites: *Teaching credential and consent of instructor.*
Reasoning about measurable characteristics in problem context. and relationships among these measurements. Additive, multiplicative reasoning, and proportional reasoning in middle grades.

Graduate Courses
Mathematics and Science Education (MTHED)

MTHED 600. Teaching and Learning Mathematics in the Early Grades (Pre-K to 4) (3)
Prerequisites: *Mathematics Education 603 or 604 and K-12 teaching experience.*
Research in teaching and learning mathematics in preschool through grade four. Innovative early childhood mathematics curricula, promising instructional practices. Assessment techniques to guide instruction.

MTHED 601. Teaching and Learning Mathematics in the Middle Grades (3)
Prerequisites: *Mathematics Education 604 and K-12 teaching experience.*
Research on teaching and learning mathematics in grades five through eight. Innovative middle grades mathematics curricula, promising instructional practices. Assessment techniques to guide instructions.

MTHED 603. Seminar on Learning Theories in Mathematics Education (3) (Syllabus)
Prerequisite: *Consent of instructor or graduate adviser.*
Application of several major learning theories (e.g. behaviorism, structuralism, radical constructivism, information processing, and sociocultural perspectives) to research on the learning and teaching of mathematics.

MTHED 604. Seminar on Teaching Issues in Mathematics (3) (Syllabus)
Prerequisite: *Consent of instructor or graduate adviser.*
Mathematics education research pertaining to teaching of mathematics. Readings chosen to bridge theory and practice divide.

MTHED 605. Algebra in the 7-14 Curriculum (3) (Syllabus)
Prerequisite: *Consent of instructor or graduate adviser.*
Curricular change in algebra, with attention to experimental curricula, to research on learning of algebra, and to influences of technology. Implications for instruction.

MTHED 606. Selected Topics in 7-14 Mathematics Curriculum (3) (Syllabus)
Prerequisite: *Consent of instructor or graduate adviser.*
Curricular change in school mathematics, to include geometry, probability, and statistics, with attention to contemporary curricula, to research on learning and teaching in those areas, and to the influences of technology. Implications for instruction.

MTHED 607. Seminar on Research in Undergraduate Mathematics Education (3) (Syllabus)
Prerequisite: *Consent of instructor or graduate adviser.*
Research in undergraduate mathematics education and its implications for teaching. Topics include research on student thinking on concepts from calculus through abstract algebra and the teaching and learning of proof.

Graduate Courses
Mathematics Courses for Mathematics Education

MATH 600. Chaos and Fractals (3)
Prerequisites: *Mathematics 320 and 510.*
Non-linear dynamics to include Cantor sets, fixed and periodic points, fractal dimensions, fractals, iterative processes, orbit diagrams, orbits, period doubling, and self-similarity.

MATH 601. Topics in Algebra (3)
Prerequisites: *Mathematics 320 and 330.*
Unique factorization domains, rings and ideals, groups, algebraic field extensions. A course designed for secondary school teachers.

MATH 602. Topics in Analysis (3)
Prerequisites: *Mathematics 320 and 330.*
Topics in analysis, including the real number system, convergence, continuity, differentiation, the Riemann-Stieltjes integral, complex analysis, designed to give the secondary teacher a broad understanding of the fundamental concepts.

CSULB Graduate Program

Prerequisite Courses

MATH 247 - Introduction to Linear Algebra
(3 units)
Prerequisite: MATH 123.
Matrix algebra, solution of systems of equations, determinants, vector spaces including function spaces, inner product spaces, linear transformations, eigenvalues, eigenvectors, quadratic forms, and applications. Emphasis on computational methods.
Both grading options. (Lecture 3 hrs.)

MATH 310 - History of Early Mathematics
(3 units)
Prerequisite/Corequisite: At least one of MATH 224 or MATH 233 or MATH 247.
History of mathematics through seventeenth century, including arithmetic, geometry, algebra, and beginnings of calculus. Interconnections with other branches of mathematics. Writing component; strongly recommended students enrolling have completed the G.E. A2 requirement.
Both grading options. (Lecture 3 hrs.)

MATH 341 - Number Theory
(3 units)
Prerequisite: A grade of "C" or better in MATH 233
Divisibility, congruences, number theoretic functions, Diophantine equations, primitive roots, continued fractions. Writing proofs.
Both grading options. (Lecture 3 hrs.)

MATH 355 - College Geometry
(3 units)
Prerequisite: A grade of "C" or better in MATH 247.
Euclidean geometry, geometric objects, isometry and similarity, transformations and symmetry, algebra and geometry of complex numbers, and topics in non-Euclidean geometry and the axioms of geometry. Writing proofs.
Both grading options. (Lecture 3 hrs.)

MATH 361A - Introduction to Mathematical Analysis I
(3 units)
Prerequisites: MATH 224 and MATH 233. Completion of 60 units.
Rigorous study of calculus and its foundations. Structure of the real number system. Sequences and series of numbers. Limits, continuity and differentiability of functions of one real variable. Writing proofs.
Both grading options. (Lecture 3 hrs.)

MATH 364A - Ordinary Differential Equations I
(3 units)
Prerequisites: MATH 224.
Prerequisite/Corequisite: MATH 247.

First order differential equations; undetermined coefficients and variation of parameters for second and

higher order differential equations, series solution of second order linear differential equations; systems of linear differential equations; applications to science and engineering.
Both grading options. (Lecture 3 hrs.)

MATH 380 - Probability and Statistics
(3 units)
Prerequisite: MATH 224.
Frequency interpretation of probability. Axioms of probability theory. Discrete probability and combinatorics. Random variables. Distribution and density functions. Moment generating functions and moments. Sampling theory and limit theorems.
Letter grade only (A-F). (Lecture 3 hrs.) Not open for credit to student with credit in STAT 380.

Upper Division Courses

A minimum of 9 graduate or *approved* upper-division units of mathematics, **including at least one 500-level mathematics course**. If not previously taken for BS or credential, this course of study must include:

- MATH 444 - Introduction to Abstract Algebra
- MATH 521 - Matrix Method in Data Analysis and Pattern Recognition
- MATH 540 - Elements of Abstract Algebra
- MATH 545 - Topics in Abstract Algebra
- MATH 550 - Elements of Topology
- MATH 555 - Topics in Topology
- MATH 560 - Functional Analysis
- MATH 561 - Elements of Real Analysis
- MATH 562 - Elements of Complex Analysis
- MATH 563 - Applied Analysis
- MATH 564 - Applied Nonlinear Ordinary Differential Equations
- MATH 565 - Topics in Real Analysis
- MATH 566 - Topics in Complex Analysis
- MATH 570 - Partial Differential Equations
- MATH 573 - Advanced Scientific Computing
- MATH 574 - Stochastic Calculus and Applications
- MATH 575 - Calculus of Variations
- MATH 576 - Numerical Analysis
- MATH 577 - Numerical Solution of Partial Differential Equations
- MATH 578 - Numerical Linear Algebra
- MATH 579 - Advanced Mathematical Modeling
- MATH 590 - Selected Topics in Mathematics

A minimum of 15 graduate units of mathematics education including:
Take both of the following:
- MTED 511 - Mathematics Teaching and Learning (3 units)
- MTED 512 - Curriculum and Assessment in Mathematics (3 units)

At least 9 units in mathematics education
At least 9 units in mathematics education chosen in consultation with the Mathematics Education Graduate Advisor from the following courses:

- MTED 540 - Algebra in the School Curriculum (3 units)
- MTED 550 - Geometry and Measurement in the School Curriculum (3 units)
- MTED 560 - Analysis in the Secondary Curriculum (3 units)
- MTED 570 - Mathematical Modeling in the School Curriculum (3 units)
- MTED 580 - Probability and Statistics in the School Curriculum (3 units)
- MTED 590 - Special Topics in Mathematics Education (1-3 units)

A minimum of 6 units of approved upper division or graduate electives from mathematics, mathematics education, statistics, or approved College of Education courses, chosen in consultation with the Mathematics Education Graduate Advisor.

Math Department Position on MAT vs. MA. Math Education

Mike Fredenberg, Ph.D.

Overview

Mathematics researchers and educators have reached the consensus that teachers require a certain degree of specialized subject matter knowledge to be effective mathematics teachers (Ball & Bass, 2000; Ball, Thames, & Phelps, 2008; Ma, 1999; Shulman, 1986; Silverman & Thompson, 2008). Furthermore, educators and researchers seem to agree upon that isolated proficiencies in mathematical facts, concepts, and procedures afford an insufficient set of skills for effectively teaching mathematics (Ball & Bass, 2000; Ball et al., 2008; Borko et al., 1992; Eisenhart et al., 1993; Gess-Newsome, 2002; Hill, Rowan, & Ball, 2005; Shulman, 1986). In regard to content knowledge in general, Shulman (1986) offered that such knowledge must go beyond an understanding of the facts and concepts of a subject, and he argued that subject matter knowledge integrates facts and concepts within the structure of the principles of a discipline. For example, a biology teacher should understand the many ways to organize the subject and be fluent in the specific language of biology.

Likewise, in a study of the knowledge of division with fractions held by prospective teachers, Ball (1990) presented a parallel viewpoint wherein she considered three criteria that composed teachers' *substantive knowledge* of mathematics: (a) correct knowledge of concepts and procedures, (b) understanding of underlying principles and meanings, and (c) knowledge about the nature of mathematics and the field in general. It is this idea of a substantive knowledge of mathematics that supports the Math Department's stance that the minimal qualifications to teach a dual enrollment math course is a MS in Mathematics, Master's in mathematics or applied mathematics OR Bachelor's in either of the above AND Master's in statistics, physics or mathematics education. Furthermore, the idea of substantive knowledge justifies the Department's position that a Teaching Mathematics MA does not meet the minimal qualifications. To fully explain the Department's perspective, this paper first provides an overview of the theoretical framework for Mathematical Knowledge for Teaching (MKT)--a

framework that has been widely adopted across the field of Mathematics Education, and Mathematics Teacher Education programs.

The model of MKT presented here is that of Hill, Ball and colleagues (Ball et al., 2008; Hill et al., 2008). Hill and others viewed MKT as the subject matter knowledge and pedagogical content knowledge that together support the effective teaching of mathematics. In other words, the MKT framework delineates among the many practices that mathematics teachers are expected to master in the classroom. Specifically, in the model of MKT proposed by Hill, Ball and Schilling (2008) subject matter knowledge is categorized by: (a) common content knowledge (CCK), the knowledge employed in teaching that relates mathematical concepts and topics as they are used in other mathematically rich disciplines such as engineering; (b) specialized content knowledge (SCK), the mathematical knowledge necessary for teaching from essential conceptual perspectives, but which is not of immediate interest to students; and (c) knowledge at the mathematical horizon, an awareness of how mathematical topics are related throughout the expanse of the curriculum.

Alternately, pedagogical content knowledge is composed of: (a) knowledge of curriculum, or an understanding of the types of support available to assist teaching mathematics at a particular grade; (b) knowledge of content and teaching (KCT), which can be thought of as knowing what teaching actions might help students overcome an encountered difficulty with mathematics; and (c) knowledge of content and students (KCS), which encompasses how students learn mathematics. In Hill, Ball and Schilling's model, subject matter and pedagogical content knowledge meet at the border of SCK, and KCT/KCS. This is demonstrated in Figure 1.

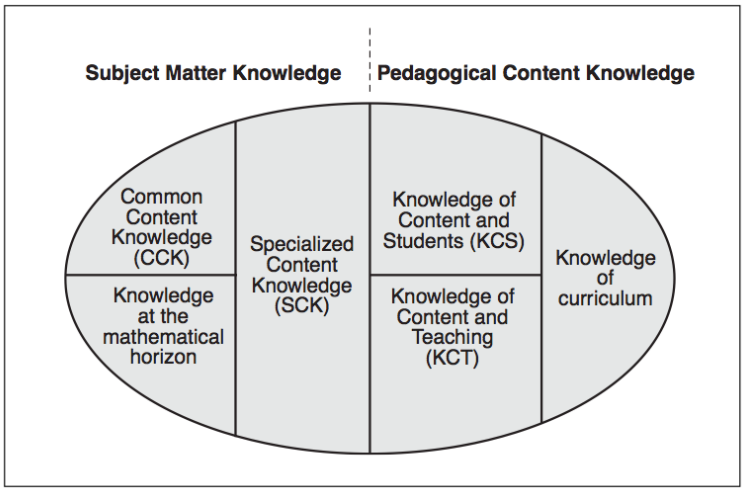

Figure 1: Components of MKT (Hill et al., 2008)

This paper focuses on the subject matter knowledge side of the model, and a short example may help clarify differences between the three subject matter content knowledge areas. Consider the two sequences, A: 2, 5, 8, 12, 15… , and sequence B: 2, 6, 12, 20, 30. These are the types of sequences that may be examined in a typical college algebra course. Two examples of Common Content Knowledge of these two sequences are the facts that sequence A is growing at a constant rate, while sequence B is not; and that sequence A can be described by a linear function, while sequence B cannot. Specialized Content Knowledge of the two sequences is knowing that the first order differences of sequence A are constant, equal to 3, and hence, 3 is the slope of the linear function. Consequently, the first order differences of sequence B are not constant, but the second order differences are constant, and this is why the second sequence can be described by a quadratic function. This knowledge of first and second order differences is not commonly known, but this knowledge is key to understanding the rich mathematics available to explore with simple sequences.

Finally, this example provides a glimpse into mathematical knowledge of the horizon—in short, where the students may be going in their mathematical academic journey. Knowledge of the first and second order differences of a sequence is essential to understanding the fundamentals of calculus, which include the concept of difference quotients, and the concepts of the derivatives of a function. The content side of the MKT model is learned in a Mathematics

MS program; it is not taught in a Teaching Mathematics MA program. This is demonstrated below.

Mathematics MS versus Teaching Mathematics MA

The MKT framework provides a useful lens through which to examine two of the main courses offered in the CSU-B Mathematics Teaching MA degree program. The two CSU-B courses reviewed in this paper are: a) MATH 6301. Algebra from a Teaching and Problem Solving Perspective; and b) MATH 6303. Advanced Concepts of Secondary Mathematics from a Teaching and Problem Solving Perspective I.

Math 6301

Here is the catalog description of the course (emphasis added by the authors):

> Algebraic thinking building from numerical reasoning, algebraic structures, and fundamental concepts of functions. **Problem solving using multiple representations will be stressed throughout**. Students will **adapt methods** from this course for implementation in a secondary classroom setting and submit a written reflection on their learning in the course.

This is a course that emphasizes pedagogy, and it belongs on the Pedagogical Content Knowledge side of the MKT model. It is not a content course that focuses on building theoretical knowledge and the understandings essential to algebraic concepts. Rather, it highlights the adaptation of methods (such as using multiple representations) to be implemented in a classroom. A course like this does not stress concepts that would be described as Specialized Content Knowledge. or Knowledge of the mathematical horizon.

Math 6303

Similarly, Math 6303:

> The first of two courses addressing concepts typically addressed in the fourth year of secondary mathematics, from an advanced perspective, with a**ctive learning teaching strategies**. Topics to include the further development of algebra, geometry, and trigonometry leading to a **problem based approach to**

> **calculus.** Emphasis on the study and application of functions. Students will **adapt methods** from this course for implementation in a secondary classroom setting and submit a written reflection on their learning in the course.

Again, this course is a teaching methods (pedagogy) course that stresses teaching strategies, and not the deep content knowledge that is necessary for meeting the minimal qualifications as a BC math instructor. For example, a problems-based approach to calculus" is fully immersed in Common Content Knowledge, and it lacks the theoretical knowledge (SKC) necessary to effectively teach a college calculus course.

In closing, a simple review of the types of courses that are offered in a MA Teaching Mathematics program provide evidence that a graduate of such a program does not meet the minimal qualifications to teach a math course for the BC mathematics department. Simply put, a graduate of a MA Teaching Mathematics program will not possess the content knowledge that is necessary to effectively teach math at the college level.

References

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-446.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. *Multiple Perspectives on the Teaching and Learning of Mathematics*. 83-104.

Ball, D. L., Sleep, L., Boerst, T. A., & Bass, H. (2009). Combining the development of practice and the practice of development in teacher education. *The Elementary School Journal*, *109*(5), 458–474.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching What Makes It Special? *Journal of Teacher Education*, *59*(5), 389–407.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education*, 23(3), 194–222.

Gess-Newsome, J. (2002). Pedagogical content knowledge: An introduction and orientation. In *Examining pedagogical content knowledge* (pp. 3–17). Springer.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4–14.

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education*, *11*(6), 499–511.